**Annotation**

of the dissertations for the degree of Doctor of Philosophy (PhD) in the specialty "6D060400 – Physics"

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# MANIFOLDS AND SUBMANIFOLD METHODS IN NONLINEAR MODELS OF GENERALIZED THEORIES OF GRAVITY

This work is devoted to the study of generalized cosmological models F(R,T,X,φ) and F(R,Q,X,φ) and their special cases by methods of differential geometry. It is shown that the symmetry of the model determines its inflationality.

**The relevance of the topic**

Constantly emerging new cosmological data leads to an understanding of the need to clarify the general theory of relativity. In particular, there is a need to determine the cause of the accelerated expansion of the universe in the initial period and in our time. In particular, the available accelerated expansion describes the k-essence model. The initial inflation, in particular, is well described by the cosmological model with F(R) gravity. As a special case, it includes a gravity model with a square term of the Ricci scalar R, which can be considered as a generalization of the general theory of relativity describing Starobinsky inflation. The disadvantage of this model is the question of the reason for the existence of the cosmological model in this form, leading to initial inflation. This question can be answered by considering the model in its most general form using the methods of symmetry theory within the framework of differential geometry.

Studies of manifolds themselves are highly relevant in mathematics and physics due to their fundamental role in modeling complex space-time structures and physical processes. The importance of manifolds is confirmed by their application in various fields of science, where they form the basis for an accurate mathematical description of physical phenomena such as gravity, quantum mechanics and others. Currently, the relevance of these studies is supported by a significant number of successful theoretical and experimental confirmations, as well as many applications in applied sciences and engineering problems.

Solutions to such problems are usually reduced to solving nonlinear differential equations. An important section of solutions of such equations within the framework of the tasks set are soliton solutions of nonlinear equations described on manifolds. Solitons described by equations such as the Korteweg-de Vries equation are consistently observed in nonlinear optical systems, hydrodynamics, and plasma physics, which confirms the scientific validity of models based on manifolds. In addition, solitons are also used in cosmology. Cosmological solitons, also called

topological defects, play an important role in theoretical models describing large-scale structures, galaxy formation, and the interaction and evolution of matter and fields in the universe. Solitons are unique nonlinear wave structures that retain their shape and energy during propagation and interaction, which makes them significant for a variety of applications in physics, engineering and mathematics. The scientific importance and relevance of solitons lies in their ability to simulate stable wave processes in various nonlinear media.

**The purpose of the work** is a theoretical study of the second-order Monge-Ampere equation describing the properties of its solution, methods of differential geometry are also used, namely:

1. To find the form of the Lagrange function for F(R,T,X,φ), F(R,X,φ), F(T,X, φ) and F(R,Q,X,φ) cosmological models by methods of Noether theory.
2. Consider the properties of the Monge-Ampere equation by methods of differential geometry, and find its solution for the case of constant curvature of the surface.
3. Compare the resulting model with astronomical observational data.

**The object of the research**

The evolution of the Universe in generalized cosmological models with k-essence.

**The subject of the study**

The effect of symmetry on the inflationary nature of the evolution of the Universe in generalized cosmological models with k-essence.

**Research methods**

Methods of differential geometry, Noether theory and Lax representation were used.

To find the Lagrangian of the model, the Noether theory method was used to identify conservation laws. The integrability of the Monge-Ampere equation was shown through the Lax representation. The Sima-Tafel method and spectral deformation were also applied to connect geometry and solitons, which made it possible to describe soliton surfaces through fundamental shapes and curvature.

**Scientific provisions for the defense**

1. The solutions of field equations for generalized F(R,T,X,φ) and F(R,Q,X,φ) cosmological models, and their particular cases F(R,X,φ), F(T,X,φ), F(Q,X,φ) in the form of solving the Monge-Ampère equation, are analogous to the Starobinsky model describing the inflationary epoch of the Universe's early expansion.
2. The Lax representation for the second-order nonlinear Monge-Ampère equation demonstrates the integrability of the obtained model within the framework of soliton theory.
3. The constant cosine of the coordinate angle describing the quadratic solution of the Monge-Ampère equation shows the correspondence of parameters with observational data.

**The novelty of the dissertation work**

1. It is shown that the inflationary model is the result of the symmetry of generalized cosmological models F(R,T,X,φ) and F(R,Q,X,φ) with k essence. It is found that a special case of the solution is the Starobinsky cosmological model.

2. It is shown that the cosmological models F(R,X,φ), F(Q,X,φ) and F(T,X,φ) also have a solution in the form of a solution of the Monge-Ampere equation, one of the solutions of which includes the Starobinsky model. In this case, the model F(R,T,φ) with a scalar field does not have such a solution.

3. It is shown that the solution for the considered models can have a soliton character.

4. It is shown that the parameters of the considered F(R,X,φ) model for slow-rolling cosmology correspond to the observational data.

**Scientific and practical significance of the work**

The dissertation work is theoretical in nature. Therefore, the scientific results obtained in the course of the study were published in the open press. Dissemination of the results of the work among potential users, the community of scientists and the general public is carried out by publishing them in the form of articles in conference collections, articles in publications of the Republic of Kazakhstan and foreign peer-reviewed journals with a non-zero impact factor. The target consumers of the research results are scientists conducting similar work in the field of the soliton surface, the theory of gravity and other branches of physics and other branches of science. Along with this, the work can be introduced in the form of academic disciplines, as well as in the form of special courses for undergraduates and PhD doctoral students at universities and research centers.

In addition, the results of the dissertation work can be applied in the educational process when reading elective courses to bachelors, undergraduates and doctoral students of the specialty "Physics".

**Reliability of the results of the work**

The results obtained are confirmed by publications in leading scientific journals with a high impact factor, as well as in journals recommended by the Committee of Education and Science of the Republic of Kazakhstan. In addition, the availability of publications in the collections of materials of domestic and foreign international scientific conferences once again confirms the reliability of the results.

**The sources of the research** are the original scientific works listed in the list of references

**The author's personal contribution**

The main results of the study were obtained by the author. Numerical calculations and graphs were found by the author independently. Publications on the topic of the dissertation work were prepared by the author personally. The formulation of tasks, the choice of the research method and the discussion of the results of the work were carried out jointly with scientific consultants.

**Publication of the results**

According to the results of the dissertation work, 9 articles were published, including 1 publication in the journal Q1, percentile 66, included in the Web of Science and CiteScore in the Scopus database percentile 88, and 3 publications included in the KKSON RK, 5 articles and abstracts in the materials of international and national conferences.

**Articles with a high impact factor in the Thomson Reuters database or publications included in the international scientific database Scopus:**

1.Inflation from the Symmetry of the Generalized Cosmological Model // Symmetry. – 2021. – Vol. 13(12). – p. 2254 (according to the Web of Science database IF=2.940, Q1, percentile 66, according to CiteScore in the Scopus database percentile 88).

**Articles in publications recommended by the Ministry of Higher Education of the Ministry of Science of the Republic of Kazakhstan:**

1. Soliton geometry using the Lax pair of isomonodromic deformation // News of NAS RK. Series Physico – mathematical. – 2021. – Vol. 3(337). – P. 2025 (included in the KKSON RK).
2. The solution of the Manakov periodic table for soliton surfaces // KazNPU. Swagger. The series "Physics-mathematical sciences". – 2019. – №2(66). – Pp. 121-126 (included in the KKSON RK).
3. Thermodynamics and Geometric Thermodynamics of Reissner-Nordström Black Holes in Multidimensional Models with Power-Law Dependence // Bulletin of KazNU. Physics Series. №2(66). Almaty, 2021. pp. 18-28.

**Publications in the materials of international and foreign scientific conferences, as well as domestic conferences**

1.Generalized F(R, T) cosmological models with fermionic fields // Journal of Physics: Conference Series. – 2021. – Vol. 2090. – p. 012063.

2.Soliton surface for complex modified Korteweg – de Vries equation // Journal of Physics: Conference Series. – 2019. – Vol. 1391. – p. 012108.

3.Investigation of a soliton surface by the methods of the Gauss equation // Proceedings of the IV International Conference «Astrophysics, gravity and cosmology» (Nur-Sultan, 2019. – P. 32-36).

4.Generalized F(R,X,φ) cosmology // Collection of materials of the XIX International Scientific Conference of students and young scientists "Gylym Jane Bilim-2024" (Astana, 2024. – pp. 246-249).

5. Study of the cosmological model by methods of the F(R, X, φ) symmetry theory // Collection of materials of the XIX International Scientific Conference of students and young scientists "Gylym jane bilim-2024" (Astana, 2024. – pp. 299-302).

**Approbation of the dissertation**

The results of the work were presented and discussed at the following domestic and foreign international conferences:

– at scientific seminars of the Department of General and Theoretical Physics of the L.N.Gumilev ENU;

- Proceedings of the IV International Conference «Astrophysics, gravity and cosmology. Astana - 2019." L.N.Gumilev ENU;

– at the International Scientific Conference of Students and Young Scientists "Gylym jane bilim-2024" of L.N.Gumilev ENU;

**Dissertation on the topic of connection with scientific work**

The work was carried out in accordance with the plans of research work on the project -"Study of the Monge–Ampere equation by geometric methods of soliton theory and its application to solving physical problems". Project years - 05/17/2023-12/31/2025. IRN project AP19175860.

**Volume and structure dissertation work**

The dissertation consists of an introduction, three sections, a conclusion and a list of sources used. The work is presented on 87 pages, contains 8 figures, 460 formulas are given, 97 names are listed.